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DETERMINING PARAMETERS AND SIMULATING THE MOVEMENT OF THE TEMPERATURE FRONT IN NON-ISOTHERMAL FILTRATION

The dynamics of objects with distributed parameters are described using partial differential equations of the parabolic type, which with boundary conditions are mathematical models of many non-stationary nonlinear processes. Systems of equations of the parabolic type with the same boundary conditions were used to build mathematical models of heat and mass transfer.

The task is set to choose the optimal method of solving this or that field theory problem and the technical means of its implementation, taking into account the nonlinearity of real processes.

In the mathematical modeling of complex objects with distributed parameters, methods of discretization of the mathematical model by spatio-temporal quantization were used. The mathematical model of objects with distributed parameters is represented by systems of ordinary differential (or algebraic) equations, which allows them to be modeled on analog and digital computers.

It is assumed that the time of operation of the circulation system is limited by the time of reaching the temperature front of the production well. The conducted studies [1] established that the heat inflow from the rock massif surrounding the layer in real reservoir conditions does not have a significant effect on the time of operation of the circulation system in a constant temperature regime. Therefore, the heat inflow is neglected in the calculations. When extracting geothermal energy, pressure filtration takes place, in which the value of μ has a value of the order of 10-6 m-2. In this connection, the system enters the stationary mode in a time that is small compared to the time of its operation.

A method of modeling the movement of the temperature front using a differential model with transition to a finite-difference model is proposed. After calculating the first approximation of the value of the speed of movement of cold water, this value was refined using iterations on various parameters of the model.

Key words: *mathematical model, temperature front, heat transfer agent.*

Modeling the movement of the temperature front. If we assume that the temperature of the liquid changes in leaps and ranges from T_{hot} – the temperature of hot water to T_{cold} – the temperature of cold water, then the boundary B of the transition from one temperature to another is a temperature front. The filtration coefficient (in general, it can be piecewise-constant, that is, depending on the coordinates) when passing through the boundary B changes from K_{hot} – the filtration coefficient of hot water to K_{cold} – the filtration coefficient of cold water.

The following method of modeling the movement of the temperature front is proposed, based on the discontinuity of the fluid flow at the boundary B

$$-K_{cold}\left.\frac{\partial H_{cold}}{\partial n}\right|_B = -K_{hot}\left.\frac{\partial H_{hot}}{\partial n}\right|_B .$$

Taking into account all that was said above and considering the power of the layer as a piecewise-constant quantity, we proceed from the original system of differential equations [2, (1)-(2)] to the next system of equations

$$K_{cold}m\frac{\partial^2 H_{cold}}{\partial x^2} + K_{cold}m\frac{\partial^2 H_{cold}}{\partial y^2} = 0, \qquad (1)$$

$$K_{hot}m\frac{\partial^2 H_{hot}}{\partial x^2} + K_{hot}m\frac{\partial^2 H_{hot}}{\partial y^2} = 0. \qquad (2)$$

Let's discretize the system of equations (1) and (2) using finite-difference schemes . Then the system of equations for node i , j in the finite-difference form will have the form:

$$\frac{K_{cold}m}{h^2}\left[\left(H_{i+1,j}^{cold} - H_{i,j}^{cold}\right) + \left(H_{i-1,j}^{cold} - H_{i,j}^{cold}\right) + \left(H_{i,j+1}^{cold} - H_{i,j}^{cold}\right) + \left(H_{i,j-1}^{cold} - H_{i,j}^{cold}\right)\right] = 0;$$

$$\frac{K_{hot}m}{h^2}\left[\left(H_{i+1,j}^{hot} - H_{i,j}^{hot}\right) + \left(H_{i-1,j}^{hot} - H_{i,j}^{hot}\right) + \left(H_{i,j+1}^{hot} - H_{i,j}^{hot}\right) + \left(H_{i,j-1}^{hot} - H_{i,j}^{hot}\right)\right] = 0.$$

Let's enter the scale $H = K_H U + H_{\min}$, we get

$$\frac{K_{cold}m}{h^2}\left(U_{i+1,j}^{cold} + U_{i-1,j}^{cold} + U_{i,j+1}^{cold} + U_{i,j-1}^{cold} - 4U_{i,j}^{cold}\right) = 0; \quad (3)$$

$$\frac{K_{hot}m}{h^2}\left(U_{i+1,j}^{hot} + U_{i-1,j}^{hot} + U_{i,j+1}^{hot} + U_{i,j-1}^{hot} - 4U_{i,j}^{hot}\right) = 0. \quad (4)$$

Kirchhoff's law for node i , j of the resistive grid will be written as follows

$$\frac{1}{R_{cold}}\left(U_{i+1,j}^{cold} + U_{i-1,j}^{cold} + U_{i,j+1}^{cold} + U_{i,j-1}^{cold} - 4U_{i,j}^{cold}\right) = 0; \quad (5)$$

$$\frac{1}{R_{hot}}\left(U_{i+1,j}^{hot}+U_{i-1,j}^{hot}+U_{i,j+1}^{hot}+U_{i,j-1}^{hot}-4U_{i,j}^{hot}\right)=0. \quad (6)$$

From equations (3)-(6), the following expressions can be obtained for grid resistances and currents simulating fluid flow

$$R_{cold}=\frac{K_R}{K_{cold}m},\ R_{hot}=\frac{K_R}{K_{hot}m},\ i=\frac{QU}{HK_{hot}mR_{hot}}.$$

The radial nature of the fluid flow near the wells is taken into account by the additional resistance, which is determined by by the formula:

$$R_{add}=R\left(\frac{1}{2\pi}l_n\frac{h}{r_{well}}-\frac{1}{n}ctg\,\tfrac{\pi}{n}\right).$$

Methodology for modeling the system of equations (1)-(2) on we will consider the grid model using the example of a five-point model schemes placement of wells, and the number of nodes between wells is not plays a role to implement the method. Due to the symmetry of the problem, 1/8 of the block is considered.

Parameters of the movement of the temperature front. The determining factor is the speed of water running along the main flow line in the direction of grid nodes $i\,,j-i+1,j+1-\ldots-i+n\,,j+n$, etc.). The speed of water movement for the specific area under consideration is determined by the formula

$$v_{hot}^{i,j;i+1,j+1}=\frac{K_{hot}\Delta U_{hot}^{i,j;i+1,j+1}K_H}{lp},$$

where $i\,,j$ are the indices of nodal points; $\Delta U_{hot}^{i,j;i+1,j+1}$ – potential difference between the corresponding nodal points; K_H – the scale of the dimensionality of the pressure function; l is the distance between the corresponding nodal points.

The rate of advance of the temperature front in this area is determined by the speed of movement of the coolant

$$v_T^{i,j;i+1,j+1}=\frac{v_{hot}^{i,j;i+1,j+1}}{f},$$

where f is a coefficient that takes into account the decrease in the speed of the temperature front in comparison with the hydrodynamic speed of the fluid's movement. Coefficient f is determined by the following dependence

$$f=\frac{c_0\rho_0(1-p)+c\rho p}{c_0\rho_0(1-p)}=1+\frac{c\rho p}{c_0\rho_0(1-p)}.$$

Then the time of movement of the temperature front along the considered area is determined by the formula

$$t^{i,j;i+1,j+1}=\frac{fl}{v_{hot}^{i,j;i+1,j+1}}.$$

speed of water running in the Y direction along the entire temperature front formed at the previous moment in time is determined by the formulas:

a) if movement in the Y direction starts from a nodal point

$$v_y^{i,j;i,j+1}=\frac{K_{hot}\Delta U_{hot}^{i,j;i,j+1}}{hp},$$

where $\Delta U_{hot}^{i,j;i,j+1}$ is the potential difference between the corresponding nodal points, h is the grid step;

b) if the movement of cold water begins between nodal points

$$v_y^{i,j;i,j+1}=\frac{K_{cold}\Delta U_y^{i,j;i,j+1}K_H\left(R_{cold,y}^{i,j;i,j+1}\right)'}{p\left(l_{cold,y}^{i,j;i,j+1}\right)'_p\left(R_y^{i,j;i,j+1}\right)'},$$

where $\left(R_{cold,y}^{i,j;i,j+1}\right)'$ is the resistance of the section of cold water between the corresponding nodal points at the previous time point along the Y axis, $\left(R_y^{i,j;i,j+1}\right)'$ is the resistance of the entire water section between the corresponding nodal points at the previous time point along the Y axis, $\left(l_{cold,y}^{i,j;i,j+1}\right)'_p$ is the distance traveled by the cold water at the previous time point.

Distances covered by the thermal front in the Y direction are determined by speed, taking into account time

$$l_{cold,y}^{i,j;i,j+1}=\frac{v_y^{i,j;i,j+1}t^{i,j;i+1,j+1}}{f}+\left(l_{cold,y}^{i,j;i,j+1}\right)'_p.$$

If we $l_{cold,y}^{i,j;i,j+1}>h$, determine the time when the thermal front reaches the point

$$t_y^{i,j;i,j+1}=\frac{f\left[h-\left(l_{cold,y}^{i,j;i,j+1}\right)'_p\right]}{v_y^{i,j;i,j+1}},$$

and

$$\Delta t_y^{i,j;i,j+1}=t_y^{i,j;i+1,j+1}-t_y^{i,j;i,j+1}.$$

Y axis

$$v_y^{i,j+1;i,j+2}=\frac{K_{hot}\Delta U_{y,hot}^{i,j+1;i,j+2}K_H}{ph}.$$

The distance traveled on this section is determined as

$$l_{cold,y}^{i,j+1;i,j+2}=\frac{v_y^{i,j+1;i,j+2}\Delta t^{i,j;i,j+1}}{f}$$

etc. Then we form a front of cold water.

According to the configuration of the thermal front at the moment of time, the resistances of the electric grid in the Y and X directions are determined by the formulas:

$$R_y^{i,j;i,j+1}=R_{cold,y}^{i,j;i,j+1}+R_{hot,y}^{i,j;i,j+1}=R_{hot}\left(\frac{K_{hot}\left(l_{cold,y}^{i,j;i,j+1}\right)_p}{K_{cold}h}+\frac{l_{hot,y}^{i,j;i,j+1}}{h}\right);$$

$$R_x^{i,j;i+1,j}=\frac{R_{cold,x}^{i,j;i+1,j}\cdot R_{hot,x}^{i,j;i+1,j}}{R_{cold,x}^{i,j;i+1,j}+R_{hot,x}^{i,j;i+1,j}}=\frac{R_{hot}\frac{K_{hot}}{K_{cold}}h}{\left(F_{cold,x}^{i,j;i+1,j}\right)_p+\frac{K_{hot}}{K_{cold}}F_{hot,x}^{i,j;i+1,j}},$$

where $R_y^{i,j;i,j+1}$ and $R_x^{i,j;i+1,j}$ are the supports of the entire section between the corresponding nodal points (at the moment of time) along the Y and X axes, respectively, $\left(l_{cold,y}^{i,j;i,j+1}\right)_p$ is the real distance occupied by cold water due to the configuration of the thermal front, $R_{cold,y}^{i,j;i,j+1}$ and $R_{cold,x}^{i,j;i+1,j}$ are the supports of the sections of cold water between the corresponding nodal points (at the moment of time) along the Y and X axes, respectively, $l_{hot,y}^{i,j;i,j+1}$ is the distance occupied by hot water between the corresponding nodal points (at the moment of time) along the Y axis, $l_{hot,y}^{i,j;i,j+1} = h - \left(l_{cold,y}^{i,j;i,j+1}\right)_p$, $\left(F_{cold,x}^{i,j;i+1,j}\right)_p$ is the cross section occupied by cold water in the section modeled by the considered X resistance (at the current moment in time), $F_{hot,x}^{i,j;i+1,j}$ is the cross section occupied by hot water in the section simulated by the considered X resistance (at the current moment in time), $F_{hot,x}^{i,j;i+1,j} = h - \left(F_{cold,x}^{i,j;i+1,j}\right)_p$.

The values of the indicated resistances on the model are changed.

The speed of water running along the main flow line in cold water is calculated according to the formula

$$\left(v_{cold}^{i,j;i+1,j+1}\right)^* = \frac{K_{cold}\left(\Delta U_{cold}^{i,j;i+1,j+1}\right)^* K_H}{lp},$$

where $\left(\Delta U_{cold}^{i,j;i+1,j+1}\right)^*$ is the potential difference between the corresponding nodal points.

If the water velocity determined from the cold water matches the water velocity determined from the hot water, then this is a valid value of the velocity and all the data obtained do not need to be adjusted. If the values of the velocities determined by hot and cold water differ from each other, then there is a certain velocity according to the formula

$$\left(v_{cold}^{i,j;i+1,j+1}\right)_1 = \frac{v_{hot}^{i,j;i+1,j+1} + \left(v_{cold}^{i,j;i+1,j+1}\right)^*}{2}.$$

Then iterations are carried out until complete coincidence $\left(v_{cold}^{i,j;i+1,j+1}\right)_n$ and $\left(v_{cold}^{i,j;i+1,j+1}\right)_n^*$, where n is the iteration number.

The procedure for carrying out iterations to determine the speed of cold water:

1.1. First iteration. Determination of the speed of water running along the main flow line.

$$\left(v_{cold}^{i,j;i+1,j+1}\right)_1 = \frac{v_{hot}^{i,j;i+1,j+1} + \left(v_{cold}^{i,j;i+1,j+1}\right)^*}{2}.$$

1.*n*. nth iteration

$$\left(v_{cold}^{i,j;i+1,j+1}\right)_n = \frac{\left(v_{cold}^{i,j;i+1,j+1}\right)_{n-1} + \left(v_{cold}^{i,j;i+1,j+1}\right)_{n-1}^*}{2}.$$

2.1. First iteration. Time it takes the water to pass the corresponding area.

$$t_1^{i,j;i+1,j+1} = \frac{fl}{\left(v_{cold}^{i,j;i+1,j+1}\right)_1}.$$

2.*n*. nth iteration

$$t_n^{i,j;i+1,j+1} = \frac{fl}{\left(v_{cold}^{i,j;i+1,j+1}\right)_n}.$$

3.1. First iteration. Determination of the speed of water running in the Y direction.

$$\left(v_y^{i,j;i,j+1}\right)_1 = \frac{K_{cold}\left(\Delta U_{cold}^{i,j;i,j+1}\right)_1 K_H R_{cold,y}^{i,j;i,j+1}}{p\left(l_{cold,y}^{i,j;i,j+1}\right)_p R_y^{i,j;i,j+1}}.$$

3.*n*. nth iteration

$$\left(v_y^{i,j;i,j+1}\right)_n = \frac{K_{cold}\left(\Delta U_{cold}^{i,j;i,j+1}\right)_n K_H \left(R_{cold,y}^{i,j;i,j+1}\right)_{n-1}}{p\left(l_{cold,y}^{i,j;i,j+1}\right)_{n-1,p} \left(R_y^{i,j;i,j+1}\right)_{n-1}}.$$

4.1. First iteration. Determination of the distances traveled by the thermal front between the corresponding nodal points.

$$\left(l_{cold,y}^{i,j;i,j+1}\right)_1 = \frac{\left(v_y^{i,j;i,j+1}\right)_1 \left(t^{i,j;i+1,j+1}\right)_1}{f} + \left(l_{cold,y}^{i,j;i,j+1}\right)'_p.$$

If $\left(l_{cold,y}^{i,j;i,j+1}\right)_1 > h$, so

$$\left(t_y^{i,j;i,j+1}\right)_1 = \frac{f\left[h - \left(l_{cold,y}^{i,j;i,j+1}\right)'_p\right]}{\left(v_y^{i,j;i,j+1}\right)_1},$$

$$\left(\Delta t_y^{i,j;i,j+1}\right)_1 = \left(t^{i,j;i+1,j+1}\right)_1 - \left(t_y^{i,j;i,j+1}\right)_1,$$

$$\left(v_y^{i,j+1;i,j+2}\right)_1 = \frac{K_{cold}\left(\Delta U_y^{i,j+1;i,j+2}\right)_1 K_H R_{cold}^{i,j+1;i,j+2}}{p\left(l_{cold,y}^{i,j+1;i,j+2}\right)_p R_y^{i,j+1;i,j+2}},$$

$$\left(l_{cold,y}^{i,j+1;i,j+2}\right)_1 = \frac{\left(v_y^{i,j+1;i,j+2}\right)_1 \left(\Delta t_y^{i,j;i,j+1}\right)_1}{f}.$$

4.*n*. nth iteration

$$\left(l_{cold,y}^{i,j;i,j+1}\right)_n = \frac{\left(v_y^{i,j;i,j+1}\right)_n \left(t^{i,j;i+1,j+1}\right)_n}{f} + \left(l_{cold,y}^{i,j;i,j+1}\right)'_p.$$

If $\left(l_{cold,y}^{i,j;i,j+1}\right)_n > h$, so

$$\left(t_y^{i,j;i,j+1}\right)_n = \frac{f\left[h - \left(l_{cold,y}^{i,j;i,j+1}\right)'_p\right]}{\left(v_y^{i,j;i,j+1}\right)_n},$$

$$\left(\Delta t_y^{i,j;i,j+1}\right)_n = \left(t^{i,j;i+1,j+1}\right)_n - \left(t_y^{i,j;i,j+1}\right)_n,$$

$$\left(v_y^{i,j+1;i,j+2}\right)_n = \frac{K_{cold}\left(\Delta U_y^{i,j+1;i,j+2}\right)_n K_H \left(R_{cold}^{i,j+1;i,j+2}\right)_{n-1}}{p\left(l_{cold,y}^{i,j+1;i,j+2}\right)_{n-1} \left(R_y^{i,j+1;i,j+2}\right)_{n-1}},$$

$$\left(l_{cold,y}^{i,j+1;i,j+2}\right)_n = \frac{\left(v_y^{i,j+1;i,j+2}\right)_n \left(\Delta t_y^{i,j;i,j+1}\right)_n}{f}.$$

5.1. First iteration. According to the configuration of the thermal water front at the moment of time, the supports of the electric grid in the Y and X directions are determined.

$$\left(R_y^{i,j;i,j+1}\right)_1 = \left(R_{cold,y}^{i,j;i,j+1}\right)_1 + \left(R_{hot,y}^{i,j;i,j+1}\right)_1 = \frac{K_{hot}\left(l_{cold,y}^{i,j;i,j+1}\right)_{p,1}}{K_{cold}h} + \frac{\left(l_{hot,y}^{i,j;i,j+1}\right)_1}{h};$$

$$\left(R_x^{i,j;i+1,j}\right)_1 = \frac{\left(R_{cold,x}^{i,j;i+1,j}\right)_1 \cdot \left(R_{hot,x}^{i,j;i+1,j}\right)_1}{\left(R_{cold,x}^{i,j;i+1,j}\right)_1 + \left(R_{hot,x}^{i,j;i+1,j}\right)_1} = \frac{\frac{K_{hot}}{K_{cold}}h}{\left(F_{cold,x}^{i,j;i+1,j}\right)_{p,1} + \frac{K_{hot}}{K_{cold}}\left(F_{hot,x}^{i,j;i+1,j}\right)_1}.$$

5.*n*. nth iteration

$$\left(R_y^{i,j;i,j+1}\right)_n = \left(R_{cold,y}^{i,j;i,j+1}\right)_n + \left(R_{hot,y}^{i,j;i,j+1}\right)_n = \frac{K_{hot}\left(l_{cold,y}^{i,j;i,j+1}\right)_{p,n}}{K_{cold}h} + \frac{\left(l_{hot,y}^{i,j;i,j+1}\right)_n}{h};$$

$$\left(R_x^{i,j;i+1,j}\right)_1 = \frac{\left(R_{cold,x}^{i,j;i+1,j}\right)_n \cdot \left(R_{hot,x}^{i,j;i+1,j}\right)_n}{\left(R_{cold,x}^{i,j;i+1,j}\right)_n + \left(R_{hot,x}^{i,j;i+1,j}\right)_n} = \frac{\frac{K_{hot}}{K_{cold}}h}{\left(F_{cold,x}^{i,j;i+1,j}\right)_{p,n} + \frac{K_{hot}}{K_{cold}}\left(F_{hot,x}^{i,j;i+1,j}\right)_{p,n}}.$$

6.1. – first iteration. The speed of water running along the main current line is recalculated.

$$\left(v_{cold}^{i,j;i+1,j+1}\right)_1^* = \frac{K_{cold}\left(\Delta U^{i,j;i+1,j+1}\right)_1^* K_H}{ph}.$$

A comparison is made with the speed obtained in paragraph 1.1 and, if there is a discrepancy between them, the average speed is determined.

$$\left(v_{cold}^{i,j;i+1,j+1}\right)_2^* = \frac{\left(v_{cold}^{i,j;i+1,j+1}\right)_1 + \left(v_{cold}^{i,j;i+1,j+1}\right)_1^*}{2}.$$

6. *n* .n is *the* iteration

$$\left(v_{cold}^{i,j;i+1,j+1}\right)_n^* = \frac{K_{cold}\left(\Delta U^{i,j;i+1,j+1}\right)_n^* K_H}{ph}.$$

A comparison is made with the speed obtained in point 1.*n* and, if there is a discrepancy between them, the average speed is determined.

$$\left(v_{cold}^{i,j;i+1,j+1}\right)_{n+1}^* = \frac{\left(v_{cold}^{i,j;i+1,j+1}\right)_n + \left(v_{cold}^{i,j;i+1,j+1}\right)_n^*}{2}.$$

Speed value is found as a result of iterations, we move on to the next time step.

Conclusions. Differential or finite-difference models make it possible to effectively determine the parameters of the movement of the temperature front in the surrounding rocks with sufficient accuracy. But in case of complication of the configuration of the front or the area of heat and mass transfer , errors in input data and calculations begin to appear, which can significantly affect the accuracy of the final result. To reduce such errors , the possibility of switching to integro-differential or integral models is being considered for the problems of modeling the movement of the temperature front.

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Фуртат І.Е., Фуртат Ю.О. ВИЗНАЧЕННЯ ПАРАМЕТРІВ ТА МОДЕЛЮВАННЯ РУХУ ТЕМПЕРАТУРНОГО ФРОНТУ ПРИ НЕІЗОТЕРМИЧНІЙ ФІЛЬТРАЦІЇ

Описано динаміку об'єктів з розподіленими параметрами з використанням диференціальних рівнянь в частинних похідних параболічного типу, які із крайовими умовами є математичними моделями багатьох нестаціонарних нелінійних процесів. Використано системи рівнянь параболічного типу з такими ж граничними умовами для побудови математичних моделей тепломасопереносу.

Поставлено задачу вибору оптимального методу розв'язку тієї або іншої задачі теорії поля і технічного засобу її реалізації з врахуванням нелінійності реальних процесів

При математичному моделюванні складних об'єктів з розподіленими параметрами використано методи дискретизації математичної моделі шляхом просторово-тимчасового квантування. Математичної моделі об'єктів з розподіленими параметрами представлено системами звичайних диференціальних (або алгебраїчних) рівнянь, що дозволяє моделювати їх на аналогових і цифрових обчислювальних машинах.

Прийнято, що час роботи циркуляційної системи обмежений часом досягнення температурним фронтом експлуатаційної свердловини. Проведеними дослідженнями [1] встановлено, що теплоприток від гірського масиву, що оточує шар, у реальних пластових умовах не виявляє істотного впливу на час роботи циркуляційної системи в постійному температурному режимі. Тому в розрахунках теплопритоком знехтовано. При добуванні геотермальної енергії має місце напірна фільтрація, при якій величина μ має значення порядку 10^{-6} *м*$^{-2}$ *. У зв'язку із цим система виходить на стаціонарний режим за час, малий в порівнянні з часом її роботи.*

Запропоновано метод моделювання руху температурного фронту з використанням диференціальної моделі з переходом до кінцево-різницевої. Після обчислення першого наближення значення швидкості руху холодної води це значення уточнено з використанням ітерацій по різним параметрам моделі.

Ключові слова: *математична модель, температурний фронт, теплоносій.*